

Making Universal Reversible Turing Machines

Kenichi Morita

Hiroshima University, Higashi-Hiroshima, 739-8527, Japan

Abstract. Studies on universal Turing machines (UTM) have a long history, and various small UTMs have been given till now [2, 3, 10–12]. Here, we investigate the problem of constructing small UTMs under the constraint of *reversibility*, which is a property closely related to physical reversibility. Let $\text{URTM}(m, n)$ denote an m -state n -symbol universal reversible Turing machine. So far, six kinds of small URTMs have been shown. They are $\text{URTM}(10, 8)$, $\text{URTM}(13, 7)$, $\text{URTM}(15, 6)$, $\text{URTM}(17, 5)$, $\text{URTM}(24, 4)$, and $\text{URTM}(32, 3)$ given in [4, 7–9]. All of them can simulate cyclic tag systems, which are universal string rewriting systems proposed by Cook [1]. Furthermore, by the methods of reducing the number of states or symbols [5, 7], $\text{URTM}(3, 36654)$, $\text{URTM}(4, 168)$, and $\text{URTM}(138, 2)$ are obtained, though their sizes are large. The reference [6] contains computer simulation results, and the description files of the URTMs. In this talk, we give a survey on these URTMs, and explain how they are designed. We also discuss open problems on URTMs.

References

1. Cook, M.: Universality in elementary cellular automata. *Complex Syst.* 15, 1–40 (2004)
2. Kudlek, M., Rogozhin, Y.: A universal Turing machine with 3 states and 9 symbols. In: *Proc. DLT 2001, LNCS 2295*. pp. 311–318 (2002)
3. Minsky, M.L.: *Computation: Finite and Infinite Machines*. Prentice-Hall, Englewood Cliffs, NJ (1967)
4. Morita, K.: Reversible computing and cellular automata — A survey. *Theoret. Comput. Sci.* 395, 101–131 (2008)
5. Morita, K.: Reversible Turing machines with a small number of states. In: *Proc. NCMA 2014*. pp. 179–190 (2014). Slide file: Hiroshima Univ. Institutional Repository, <http://ir.lib.hiroshima-u.ac.jp/00036075>
6. Morita, K.: Constructing small universal reversible Turing machines. Hiroshima Univ. Institutional Repository, <http://ir.lib.hiroshima-u.ac.jp/00036736> (2015)
7. Morita, K.: Universal reversible Turing machines with a small number of tape symbols. *Fundam. Inform.* 138, 17–29 (2015)
8. Morita, K.: Two small universal reversible Turing machines. (submitted for publication)
9. Morita, K., Yamaguchi, Y.: A universal reversible Turing machine. In: *Proc. MCU 2007, LNCS 4664*. pp. 90–98 (2007)
10. Neary, T., Woods, D.: Four small universal Turing machines. *Fundam. Inform.* 91, 123–144 (2009)
11. Rogozhin, Y.: Small universal Turing machines. *Theoret. Comput. Sci.* 168, 215–240 (1996)
12. Woods, D., Neary, T.: The complexity of small universal Turing machines: a survey. *Theoret. Comput. Sci.* 410, 443–450 (2009)